J. C. BOSE'S DOUBLE-PRISM EXPERIMENT USING SINGLE PHOTON STATES VIS-A-VIS WAVE-PARTICLE DUALITY

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We discuss in this article the significance of the double-prism experiment J. C. Bose had performed in 1897 in order to verify the wave-like nature of electromagnetic radiation, using the phenomenon of optical tunneling that was predicted by Maxwell's equations. In particular, this article focuses on bringing out the contemporary relevance of Bose's double-prism setup that has turned out to be quite fruitful by using single photon states, enabling new insights into the nature of wave-particle duality in the context of quantum physics.

Introduction

Compared to his many other seminal contributions, the fundamental importance of the double-prism experiment performed by J.C. Bose is often glossed over. However, it is worth remebering that, subsequent to Hertz's experiment, Bose's experiment in 1897 was the one which provided one of the earliest empirical corroborations of the Maxwellian description of electromagnetic (EM) waves that predicted the phenomenon of tunneling– a hitherto unexplored wave-like feature which, as studied in Bose's double-prism experiment, arises essentially from the application of Maxwell's equations and the relevant boundary conditions. In this article, in Section II, we discuss this experiment by Bose from a proper historical perspective and we point out its essential features.

In recent times, an interesting implication of Bose's experiment has emerged by considering its quantum optical treatment in terms of single photon states. However, in order to appreciate the significance of this quantum analogue, it is crucial to understand the role played by single photon states. This is explained in Section III, in conjunction with a discussion of those experiments that provided the crucial impetus for studying the quantum analogue of Bose's double-prism experiment. In Section IV, we focus on the quantum optical treatment of Bose's experiment in terms of single photon states, and we explain the way in which such an experiment provides a new twist to wave-particle duality by the display of *simultaneous* particle and wave-like attributes of photons.

It is striking that Bose's two-prism experiment whose original role was to provide an experimental vindication of the intrinsic wave-like nature of EM radiation, has in the modern times acquired a wider significance by exhibiting a strong form of wave-particle duality in terms of a conceptually intriguing concomitant wave and particle-like behaviour. This serves as a counterexample of the tenet of 'mutual exclusivity' (considered to be a key ingredient of Bohr's complementary principle) as applied to the use of wave and particle-like concepts in describing the quantum phenomenon of tunneling of single photon states.

J. C. Bose's Double-prism Experiment

A. Historical background : Let us briefly recapitulate the key milestones in the history of development of the theories of light and electromagnetism. Optics formally took birth with the discovery of Snell's laws in 1637, followed by Newton's corpuscular theory and Huygen's wave theory in the same century, Young's interference experiments in 1807, leading to Fresnel's diffraction studies a decade later that firmly established the superiority of the wave theory of light over the corpuscular theory. On the other hand, in 1600, William Gilbert had initiated studies on electricity and magnetism with his demonstration of the attraction between magnets and the generation of electrostatic effects through friction. Franklin and Coulomb further developed

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the studies on electrostatic and magnetostatic phenomena in the eighteenth century, while the connection between electricity and magnetism was first suggested by Oersted in 1820 with his discovery of the deflection of a compass needle due to a changing current in an electrical circuit. Subsequently, Faraday, Lenz and Ampere contributed to a rapidly growing collection of experimental findings which were encapsulated within a concise mathematical framework by James Clerk Maxwell[1] through the discovery of his famous equations. Among other results, Maxwell had predicted the existence of electromagnetic (EM) waves travelling at the speed of light in free space, thereby unifying the two fields of optics and EM theory.

The first experimental verification of Maxwell's theory was performed by Heinrich Hertz[2]. Hertz demonstrated the propagation of electromagnetic waves through space, measured their speed of propagation, their wavelength, and also showed their identity with the behaviour of visible light through experiments on reflection and refraction. But a crucial point, relevant to this article, is that Hertz did not study the wave-like phenomena like interference and tunneling using EM radiation.

It is in this context that the two-prism experiment of J.C. Bose[3] was of special significance in verifying the essentially wave-like phenomenon of tunneling, associated with the evanescent EM waves as predicted by Maxwell's equations in the context of internal reflection.

B. Internal Reflection and Evanescent Waves from Maxwell's theory : In relation to the phenomenon of internal reflection, we consider a plane EM wave incident on the boundary between two isotropic media of refractive indices n_1 and n_2 (see Fig.1). Let the boundary be the yz

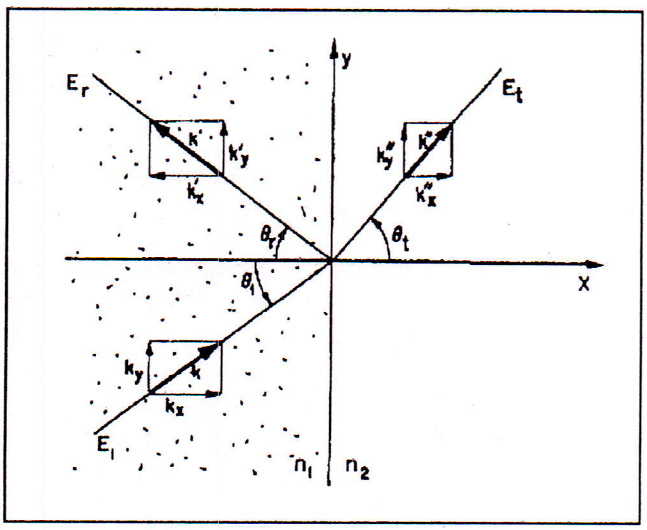

Fig. 1. The propagation vectors k, k′, k″ for the incident, reflected, and transmitted waves.

plane, and it is the medium 1 from which light is incident on the boundary between the two media. The reflected wave remains in medium 1, and the transmitted wave propagates in medium 2, while both the waves have their wave-vectors on the plane of incidence (the xy plane).

Let the electric field of the incident wave be given by $\mathbf{E}_i = \mathbf{E}_0 \exp i(\omega t - \mathbf{k}.\mathbf{x})$, where E_0 is the vector amplitude, ω the frequency and $x \equiv (x, y, z)$, $\mathbf{k} \equiv (k_x, k_y, 0)$, with $\mathbf{k}.\mathbf{k} = k^2 = (\omega^2 n_1^2)/c^2$. On application of Maxwell's equations in the regions 1 and 2 as well as at the boundary, one obtains the reflected wave given by $\mathbf{E}_r = \mathbf{E}_0' \exp i(\omega t - \mathbf{k}'.x)$ and the transmitted wave given by $\mathbf{E}_t = \mathbf{E}_0'' \exp i(\omega t - \mathbf{k}''.x)$ with the following properties (for more mathematical details, see Sommerfeld[4]). The magnitudes of the amplitudes E_0' & E_0'' are proportional to the incident amplitude E_0, with the proportionality factor depending on the angle of incidence θ_i, the two indices of refraction n_1 n_2, and on the polarisation of E_i. The wave vector k' for $\mathbf{E}_r$ has its x-component equal and opposite to that of **k**, the y component being k_y. For the transmitted wave, the y component of k'' is again k_y, while k_x'' is given by

$$k_x''^2 = \frac{k^2}{n_1^2}\left(n_2^2 - n_1^2 \sin^2\theta_i\right) \tag{1}$$

Now, let us take the medium 1 to be glass with the refractive index n, and the medium 2 to be air with the refractive index 1. Then Eq. (1) reduces to

$$k_x''^2 = \frac{k^2}{n^2}\left(1 - n^2 \sin^2\theta_i\right) \tag{2}$$

The angle of transmission θ_t is given by sin $\theta_t = n$ sin θ_i, and it becomes 90° (i.e, the transmitted wave just grazes the boundary) at the *critical angle* θ_c. For θ_i greater than the critical angle, Eq. (2) tells us that $k_x''^2$ is negative and thus k_x'' is pure imaginary. Physically, this means attenuation of the transmitted amplitude with increasing x. This wave, called the '*evanescent wave*', is represented by

$$\mathbf{E}_t = \mathbf{E}_0'' e^{-\frac{2\pi}{\lambda}\sqrt{\sin^2\theta_i - \frac{1}{n^2}}\,x} e^{i(\omega t - k_y y)} \tag{3}$$

The damping factor in the exponent is inversely proportional to the wavelength λ of the incident wave in free space and increases with the angle of incidence. For a given θ_i, the evanescent wave has negligible amplitude in air for distances beyond the boundary which are greater than λ. For further discussions, see Ref.5.

C. Bose's detection of the evanescent wave : The experimental demonstration of such a predicted evanescent wave had posed a formidable challenge since its derivation from Maxwell's theory[4]. At that time, there was no receiver for EM radiation that was sufficiently sensitive for detection, as well as capable of providing an accurate quantitative measure of the wave amplitude. Another complication was that it was very difficult to maintain a constant intensity of the source. J.C. Bose was able to circumvent both the above difficulties. In his experiment, two glass prisms were used, the first of which intercepted the incident wave as a beam splitter, while the second prism helped in amplifying the exponentially decaying evanescent wave amplitude. With this ingenious setup, Bose was able to provide a conclusive evidence of the existence of evanescent waves[3]. He verified that the dependence of the wavelength of the incident wave and its angle of incidence on the amplitude of the evanescent wave was in accordance with Eq. (3). The typical wavelengths of the EM radiation emitted by the sources used in Bose's double-prism experiment were of the order of 10 mm.

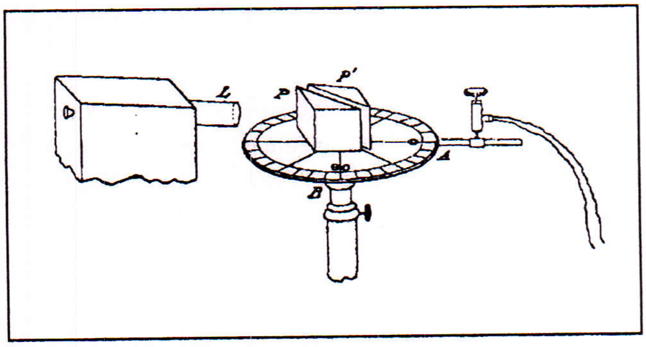

Fig. 2. L is the lens to render in incident beam parallel to the prism table ; P, P′, are the right-angled isosceles prisms; A and B are two positions of the receiver.

In order to test the effect of the angle of incidence, Bose had placed two semi-cylinders of glass on a spectrometer circle, their plane faces being separated by a suitable air-space. He placed the radiator (oscillatory discharge) at the focus of one semi-cylinder, and the evanescent wave which emerged into the air-space was focused by a second semi-cylinder on the receiver. The critical angle for glass was found to be 29°. For an angle of incidence of 30°, Bose started with an air-space of 2 cm and found no detection. By decreasing the air-space to about 13 mm, a small but measurable portion of the transmitted wave was detected. For an angle of incidence of 45°, the advent of transmission was for an air-space thickness of about 10.3 mm. This, of course, agrees with the dependence on the factor sin θ_i in the exponent of the damping factor in Eq. (3).

Next, to probe the influence of wavelength, Bose first placed a cube of glass on the prism table, and as expected saw that radiation striking one face normally is transmitted without deviation. Then, he cut the cube into half, thereby producing two prisms and kept them separated with their hypotenuses parallel. The angle of incidence was kept fixed at 45°. The receiver placed at position A (see Fig. 2) recorded the transmitted wave, while the one placed at position B recorded the reflected wave. When the two prisms were separated by distances greater than 10.3 mm, no signal was noted at position A. On reducing this distance to 10.3 mm, Bose found signals at both the positions. On further reducing the thickness, he found the reflected component decreasing and the transmitted component increasing. When the air-space was reduced to about 0.3 mm, no reflected component could be found. Bose then varied the source by gradually increasing the wavelength. He found the *minimum* thickness of air-space required for total internal reflection (or, the *maximum* thickness up to which detection of the evanescent wave was possible) to increase with increasing wavelength. This increase of the critical air gap with the increasing wavelength was again in accordance with the fact that the damping factor of the evanescent wave is inversely proportional to wavelength (see Eq. (3)).

This completes our brief discussion of Bose's double-prism experiment in its original form. Next, as already indicated in the introductory section, we proceed to discuss an instructive direction of study that has emerged in recent times leading to a quantum analogue of this experiment that throws new light on the quantum mechanical issue of wave-particle duality in relation to Bohr's complementarity principle.

The Significance of Single Photon States and the Related Experimental Studies

Subsequent to the pioneering experiment by G. Taylor[6], a number of experiments have corroborated that using extremely low-intensity pulses of light one can produce classical wave-like intereference pattern which remains the *same*, irrespective of how low the light intensities are. A common feature of all these experiments is the use of light pulses emitted from sources such as the discharge lamps and thermal sources. However, as the studies in quantum optics have revealed, these sources emit light in states (the so-called 'classical' or 'semi-classical' states) with which there is no possibility of observing single particle-like behaviour. Thus, these experiments do *not* demonstrate that 'a photon interferes with itself,' since no photon-like feature can be exhibited using such sources of light[7].

In order to observe the truly single particle-like behaviour using light pulses, one needs sources emitting what are quantum optically known as '*single photon states*' of light. These are Fock space states that are eigenstates of the 'photon number operator' corresponding to the eigenvalue unity. The probability of a joint detection of more than one photon is exactly zero for an 'ideal' single photon state. It is in this sense that single photon states entail genuine single particle-like behaviour, and therein lies the significance of single photon states[7]. For all other states (classical or nonclassical states such as multiphoton Fock states, squeezed states, or states having sub-Poissonian character), the probability of a double detection is *different* from zero even when the average number of photons (computed by expanding the relevant state as a superposition of photon number eigenstates) is less than unity. This naturally leads to the question as to whether self-interference and wave-particle duality of genuine single photon states can be experimentally demonstrated. That possibility was realized in the experiments of Aspect *et al.*[8] that served as a prelude to the quantum analogue of J. C Bose's optical tunneling experiment in terms of single photon states. We will now explain the basic argument underlying the experiments of Aspect *et al.*

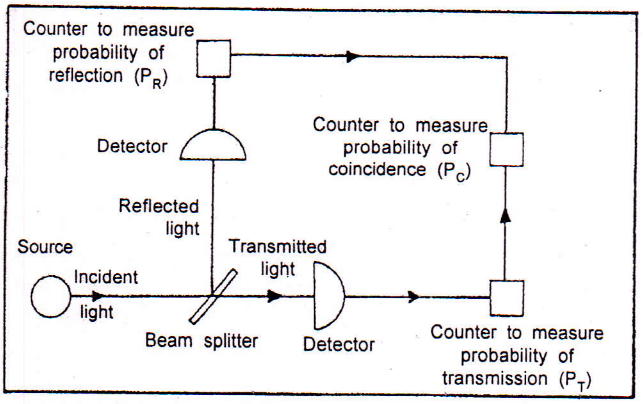

Fig. 3. An experimental arrangement to test *particle-like propagation* of single photon states.

Let us consider a source emitting light pulses well separated in time and impinging on a beam splitter (Fig. 3). Synchronized with each pulse, there is a triggering arrangement which enables the detectors to operate during an electronic gate of duration T. During that gate, single detections are monitored in the transmitted or reflected channels, and a coincidence is counted if both channels register a count during the same gate. Let the probabilities for a single count during a gate be P_t and P_r in the transmitted and reflected channels respectively, and let the probability for a coincidence during the same gate be P_c. Following Aspect and Grangier[9], one can derive a relationship between P_c, P_t, and P_r that is the characteristic of any wave-like description, based on the following general assumptions:

(a) An incident wave is split on a beam splitter; the impinging intensity I(t) being divided into a reflected and a transmitted part.

(b) I(t) is a positive quantity.

(c) Transmissivity and reflectivity of a beam splitter are constant quantities; even if they fluctuate, they can be replaced by time-averaged mean values.

(d) The probability of detection is proportional to the impinging intensity.

It follows from these assumptions that the detection probabilities P_c, P_t, and P_r can be written as follows

$$P_t = \alpha_t \eta T\langle I_n\rangle, \quad P_r = \alpha_r \eta T\langle I_n\rangle, \quad P_c = \alpha_t \eta^2 T^2 \langle I_n^2\rangle \tag{4}$$

where the bracket < > denotes an average over the ensemble of gates, α_t and α_r are transmissivity and reflectivity of the beam splitter, η is the detection efficiency, and the time-averaged intensity I_n during the nth gate opened at the instant t_n for a duration T is given by

$$I_n = \frac{1}{T}\int_{t_n}^{t_n+T} I(t)dt \tag{5}$$

Then, using the standard form of the Cauchy-Schwartz inequality, we have

$$\langle I_n^2\rangle \geq \langle I_n\rangle^2 \tag{6}$$

From Eqs. (4) and (6) one can thus obtain

$$P_c \geq P_t P_r \tag{7}$$

which gives a *lower bound* to the number of coincidences expected in any description of the *propagation* of light using a *wave-like* description. On the other hand, quantum optics predicts $P_c = P_t P_r$ for strongly attenuated pulsed laser sources, $P_c = 2P_t P_r$ for thermal sources, while for "ideal" single photon states $P_c = 0$.

At this stage, it is useful to stress that discrete localized detection events *per se* do not necessarily imply any particle-like property of the detected entities. Instead, they can be regarded as originating from the quantized energy levels of the atoms constituting the detector. Thus, the crucial point about the experiment of Fig. 3 is that the predicted anticoincidence ($P_c = 0$) for *single photon states* is a definitive signature of *single particle-like propagation* (a photon is not split on the beam splitter), in contrast to mere discrete detection subsequent to wave-like propagation. It should be noted that all other nonclassical states of light such as multiphoton Fock states, squeezed states, or states having sub-poissonian character do *not* exhibit such single particle-like propagation characteristics, and this accounts for the unique quantum character of single photon states.

Having thus explained the basic significance of single photon states, we next discuss the way Aspect *et al.*[8] had pioneered the use of single photon states in order to study wave-particle duality. For this, the crucial step was to work with a source producing *genuine* single photon states.

Familiar sources emit light by the excitation of many atoms. The atoms are excited at random times and the number of excited atoms fluctuates. Hence the emitted light is described by a density matrix that takes into account these fluctuations, including the possibility that several atoms are excited simultaneously. The single photon character is then lost. In order to see the single photon behaviour, it is therefore necessary to isolate single atom emission in space as in the experiment by Kimble *et al.*[10], or, in time , which was realized in the source used by Aspect *et al.*

The source used in the experiment by Aspect *et al.* was composed of calcium atoms in a moderate-density atomic beam, excited to the upper level of a two-photon radiative cascade, emitting two photons at different frequencies ν_1 and ν_2. An excited atom first emits a photon, say, of frequency ν_1, and goes to an intermediate metastable state of lifetime $\tau = 4.7$ns. By ensuring that the excitation rate N_e of the cascades is small enough (by suitably choosing the density of the atoms) so that $N_e \times 2\tau << 1$, once can then have cascades well separated in time. Under this condition, during any single gate, the probability for the detection of a photon ν_2 coming from the same atom that emitted ν_1 is much larger than the probability of detecting a photon ν_2 coming from any other atom in the source. In this way, a close approximation to the ideal situation of working with single photon states was achieved.

When the experiment of Fig. 3 was performed with such a source of single photon pulse, an unambiguous violation of the inequality (4) was observed. This confirmed the single particle-like behaviour of the single photon states produced in the laboratory. Then the next step was to observe interference phenomenon with the same light source, which would provide an evidence of genuine *"single photon interference."*

To test this, the same source and the same beam splitter were used, but the detectors on either side of the beam splitter were removed and the two beams were recombined using mirrors and a second beam splitter (Fig. 4). The detection rates were measured on either side of the second beam splitter - the counts which showed an interference effect that depended on the difference of path lengths along the two possible routes of the single photon pulses (the path difference was controlled by moving the mirrors).

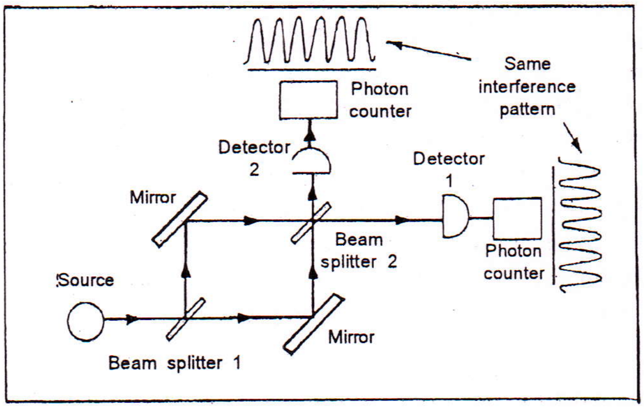

Fig. 4. An experimental arrangement to test *wave-like propagation* of single photon states.

The combination of two experiments of Figs. 3 and 4 was, therefore, able to provide an unambiguous evidence of wave-particle duality that essentially pertained to a reasonably genuine single photon state - a feature that had earlier defied a conclusive demonstration. In this context, if one follows *Bohr's complementarity principle* (henceforth, abbreviately BCP) to interpret these experiments, a particle-like model of light is relevant to the experiment of Fig. 3, while a wave model is necessary to explain the experiment of Fig. 4. The incompatibility between these two descriptions of light is circumvented in BCP by crucially contending that they are *mutually exclusive* in the sense that both the models are not required at the same time to interpret the same experiment. The experiments of Figs. 3 and 4 confirm this idea of 'mutual exclusiveness' because these two experiments cannot be performed simultaneously.

Quantum Treatment of J. C. Bose's Double-prism Experiment and Bohr's Complementarity Principle

It is against the above backdrop of experiments probing wave-particle duality and BCP using single photon states that the formulation of quantum analogue of Bose's two-prism experiment acquires a special significance because it can be used to contradict the tenet of 'mutual exclusiveness' that is considered to be the cornerstone of BCP. Here, for the sake of historical completeness, we may note Bohr's original statement about the term complementarity between wave and particle behaviours, viz. "to denote the relation of mutual exclusion characteristic of the quantum theory with regard to an application of the various classical concepts and ideas"[11]. Before going into the specifics of the quantum tunneling of single photon states using the two-prism experiment and its significance as regards BCP, it is relevant to note that Scully, Englert, and Walther[12] had given a comprehensive argument showing that, as far as the interference type experiments are concerned, the quantum mechanical formalism guarantees the validity of 'mutual exclusiveness' between the wave and particle models - this is essentially because the quantum formalism contains a built-in mechanism that ensures disappearance of the interference pattern whenever one has 'which path' information .

Like interference, *tunneling* is also a hallmark of wave-like behaviour of light for which, however, 'mutual exlusiveness' between wave and particle-like properties is not automatically enforced by the quantum mechanical formalism. It is this feature that motivated the study of quantum analogue of Bose's two-prism experiment by Ghosh, Home and Agarwal[13]. An experimental arrangement (Fig. 5) was considered similar to that used by J. C.Bose, but with the crucial feature that *single photon states* were considered for describing the light pulses incident on the combination of two prisms with a variable gap between them that allows for tunneling across any gap less than the wavelength. The quantum optical analysis of such an experiment[13] showed the possibility of *tunneling* of single photon states, crucially coexisting with *perfect anticoincidence* between the counts registered by the two detectors (1 and 2) placed behind the two prisms respectively.

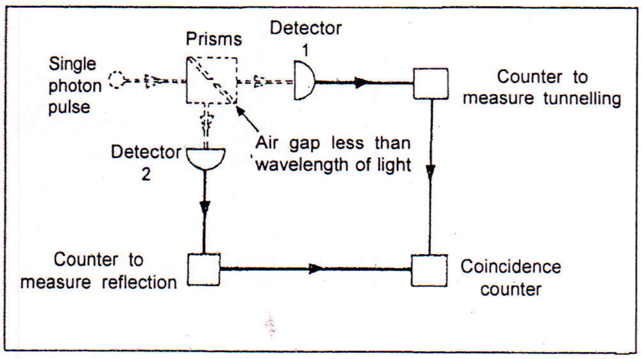

Fig. 5. A single experimental arrangement to display both classical wave and particle-like propogation of single photon states of light.

As in the case of an ordinary beam splitter, the state vector of an emergent single photon state from the two-prism arrangement, coupled with the vacuum states, can be written in the form

$$|\Psi\rangle = \alpha|1,0\rangle + \beta|0,1\rangle \tag{8}$$

where $|\alpha|^2$ gives the transmission probability and $|\beta|^2$ the reflection probability. Perfect anticoincidence between the detectors 1 and 2 (Fig 5) follows from this form of $|\Psi\rangle$.

In the quantum optical treatment of this example (some relevant details given in Apendix A), as long as there are no losses or no thermal photons added by the prism material, Maxwell equations with the classical fields are appropriately represented by using the relevant quantum mechanical operators. Then classical boundary conditions become the boundary conditions for the electric and magnetic field operators, and tunneling identical to that obtained from classical electromagnetic theory is predicted for a single photon state. Here we may stress that while anitcoincidence is a kinematic feature (implied by the structure of the state vector(1)), tunnelling follows from the dynamics of field propagation, and, importantly, the kinematic and the dynamic aspects are concomitant (instead of being mutually exclusive) in this particular setup.

Thus, this is an experimental arrangement where the observed results would contain one subset of data comprehensible in terms of a wavelike propagation, coexisting with another subset of data interpretable using a particlelike propagation embodying which-path information all the way from the source to the detectors. This form of wave-particle duality was not envisaged in Bohr's hypothesis of mutual exclusivity contained in the statement of the complementarity principle. Also, note that a key aspect of such an experiment is that anticoincidences (recording the particlelike signature) as well as the singles rates (registering the evidence of wavelike tunneling) pertain to the *same* ensemble of photons incident on the two-prism arrangement. Of course, a variant of the beam splitter experiment may also be formulated analogous to the two-prism experiment; for example, by varying the orientation of the beam splitter (i.e., by changing the angle of incidence of the incident light pulse), transmission and reflection probabilities can be varied, which is a wave-like feature. However, this effect would not be so pronounced as the one due to variations of the gap between the two prisms.

The experimental verification of the quantum optical prediction for the double-prism setup using single photon states was achieved by Mizobuchi and Ohtake[14] at the Central Research Laboratory, Hamamatsu Photonics, Japan. For this purpose, in order to produce single photon states, they used the parametric down conversion technique and appropriately selected the photons passing through the double-prism setup. In their experiment, the incident photons were of the wavelength about 350 nm, and the gap between the two prisms was kept approximately1/10th of this wavelength. Both the reflected and the transmitted photons were detected by highly efficient avalanche photodiode single-photon dectectors, along with their anticoincidence being registered with an appropriate resolving time determined by the input pulse duration.

To summarize, in the two-prism experiment involving single photon states, one obtains results consistent with both the wave-like behaviour and particle-like which-path information. This experiment, thus, confronts Bohr's waveparticle complementarity by providing an example, allowed by the quantum mechanical formalism, where the notion of mutual exclusiveness of wave and particle models ceases to be valid. This, therefore, shows that the putative generality of Bohr's wave-particle complementarity is not wholly incorporated within the quantum mechanical formalism[15,16].

Concluding Remarks

During the last few years, more studies have been pursued related to the quantum analogue of Bose's double-prism experiment and its implications for wave-particle duality. In particular, statistically more precise experimental demonstration of concomitant particle and wave-like behaviour has been achieved for a suitable variant of such a setup by using more efficient single-photon detectors, as well as by using a tunable source of single photon states[17,18]. Since wave-particle duality is one of the key quantum enigmas whose understanding provides valuable insights into the nature of quantum reality, it is not surprising that the conceptual implications of the results of such experiments continue to be much debated. Suggestions have also been made to probe further interesting features of wave-particle duality that could be uncovered by probing the quantum analogue of Bose's double-prism experiment using non-classical states of light (like the squeezed states), apart from using the single photon states. To conclude, it is indeed remarkable the way J. C. Bose's idea of conceiving such a setup has turned out to play such a significant role in the modern studies on wave-particle duality in the quantum world.

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Appendix A

Let the incident, reflected and transmitted field amplitudes be denoted by a, d and c respectively. In classical electrodynamics these field amplitudes obey the relations

$$d = \gamma a, \quad c = \alpha a \tag{A1}$$

In the quantum treatment, the quantities d, c, and a are taken as annihilation operators. Moreover, in order to maintain the commutation relations, we have to add the vacuum field b at the open port. Thus, Eq. A1 is to be modified to

$$c = \alpha a + \beta b, \quad d = \gamma a + \delta b \tag{A2}$$

and one has the commutation relations

$$\left[a, a^\dagger\right] = \left[b, b^\dagger\right] = 1, \ \left[a, b^\dagger\right] = 0, \ \left[c, c^\dagger\right] = \left[d, d^\dagger\right] = 1 \tag{A3}$$

Here $|\alpha|^2 + |\gamma|^2 = 1$, since the prisms are supposed to be lossless. The parameter β is related to γ by a phase factor at most. The probability $P_d(1)(P_c(1))$ of detecting a photon at the detector $D_1(D_2)$ is given by

$$P_d(1) = Tr\left\{P|1\rangle_d \ {}_d\langle 1|\right\}. \tag{A4}$$

where $|1\rangle_d$ is the single photon state associated with the mode *d*. Assuming the input states as $|1\rangle_a |0\rangle_b$, these probabilities can be calculated as

$$P_d(1) = |\gamma|^2, \quad P_c(1) = |\alpha|^2 \tag{A5}$$

Note that the results (A5)are the same as that can be obtained on the basis of classical electrodynamics. In order to see the essentially quantum feature, let us see if the detectors click in coincidence or anitcoincidence. The joint probability $P_{cd}(1, 1)$ of detecting one photon at D_1 and one photon at D_2 is given by

$$P_{cd}(1, 1) = Tr\left\{P|1\rangle_c |1\rangle_d \ {}_d\langle 1|_c\langle 1|\right\}. \tag{A6}$$

Using Eq. (A2), Eq. (A6) reduces to

$$P_{cd}(1, 1) = 0 \tag{A7}$$

which implies that the two detectors should always click in anticoincidence. It is thus shown that the possibility that the 'tunneling' phenomenon occurs with the two counters (1 and 2) clicking in perfect *anticoincidence* is the one that follows from the quantum optical treatment which can imply the possibility that the 'tunneling' occurs with the two counters (1 and 2) clicking in *coincidence*, only if either of these two conditions is satisfied: (i) the incident field contains more than one photon, i.e. the probability that the incident field has more than one photon is nonzero, or, (ii) the medium adds a noise photon, say, from thermal fluctuations.