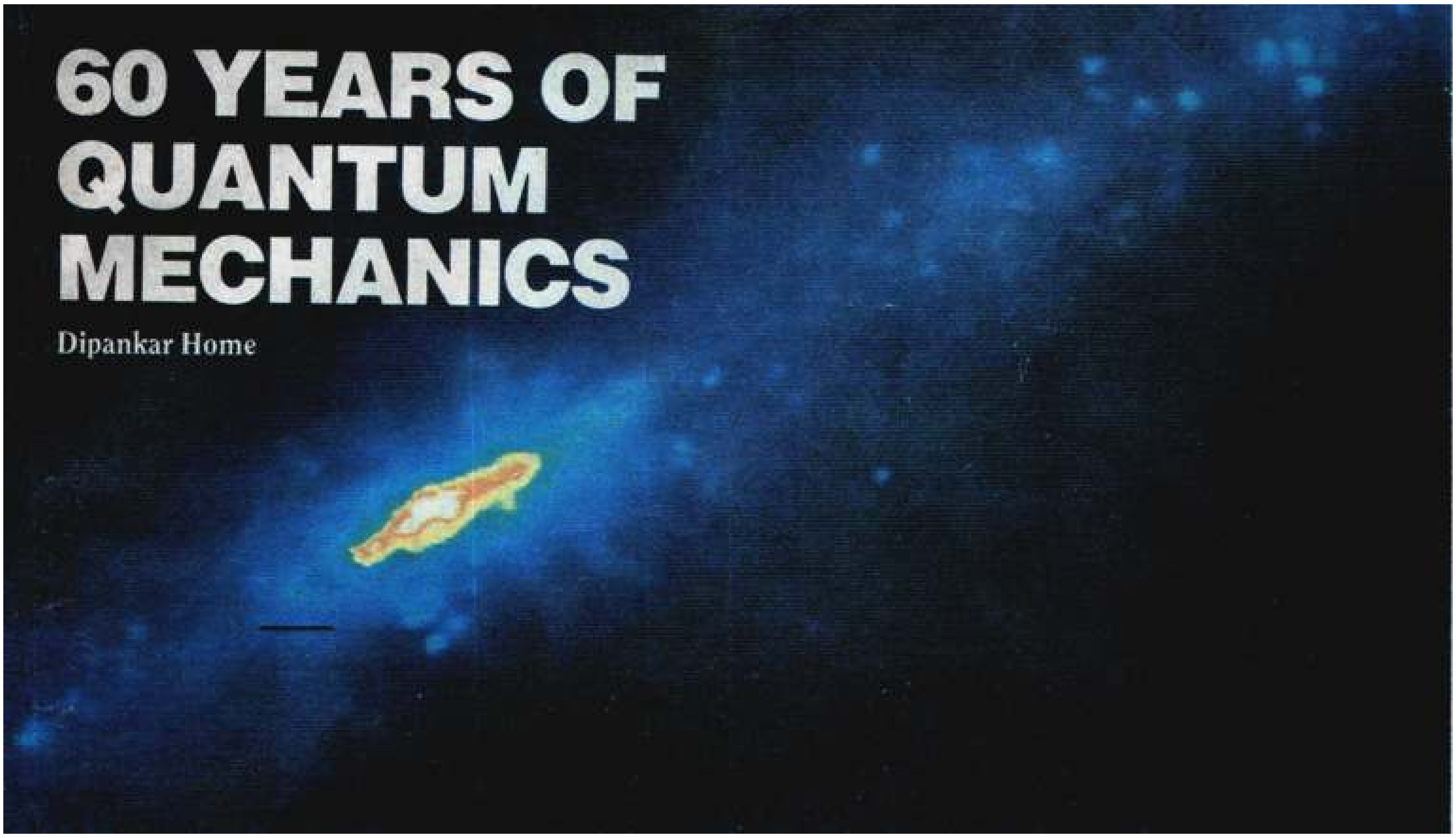
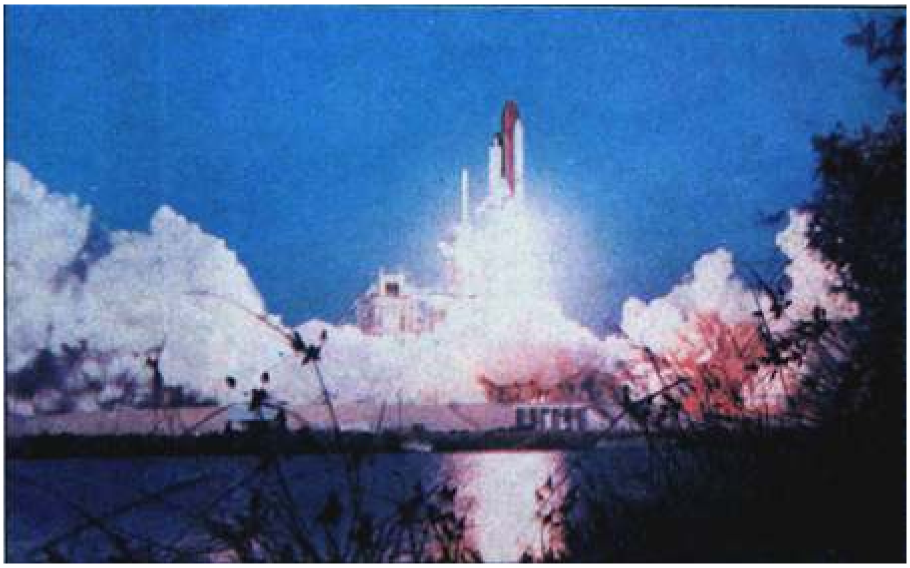
60 YEARS OF QUANTUM MECHANICS

Dipankar Home



IN June 1925, while recuperating after an attack of hay fever on the island of Heligoland in the North Sea, Werner Heisenberg--then a young research assistant to max Born at Gottingen in Germany - conceived the crucial ideas which triggered a decisive breakthrough in our understanding of the physics of atomic and sub-atomic world. On July 11, Heisenberg gave Born the final manuscript of his historic paper entitled "Quantum Theoretical Re-interpretation of Kinematic and Mechanical Relations". In a letter to Einstein dated July 15, Born remarked: "Heisenberg's latest paper appears rather mystifying but is certainly true and profound." That signalled the advent of a new system of mechanics for the atomic phenomena, called quantum mechanics, tinged with an aura of mystery.

Rarely in the History of science has there been a theory as quantum mechanics which calls for such a drastic revision of the seminal kernels of the traditional philosophy of science. At present, quantum mechanics has permeated the fabric of modern physics. Yet the sense of enigma linked with quantum mechanics persists undiminished across a span of nearly six



The advent of quantum mechanics has profoundly affected the course of 20th century science–not only physics but other disciplines like biology, medicine, engineering--resulting in many spin-offs--from the charting of distant galaxies [seen (top) through the helium-chilled eyes of the infrared Astronomical Satellite (IRAS)]; computerisation of Beethoven's Fifth Symphony (above) and (ridght launching of the Space Shuttle decades, notwithstanding its stupendous success in accounting for a staggering variety of empirical facts concerning all known atomic phenomena. We Shall be soon celebrating the sixtieth anniversary of Heisenberg's discovery. On this occasion, let us take a retrospective look with historical perspective on the genesis of the work and aim at some appreciation of the far-reaching implications sprouting from one of the greatest intellectual triumphs of human mind.

The physical universe around us is made up of matter and radiation. The primary objective of physics is to probe the underlying principles governing behaviour of matter and radiation and to elucidate how all observable physical phenomena can be understood on the basis of these principles. By the end of the nineteenth century it appeared that the physicists had gained more-or-less satisfactory grasp of the physical events occurring on the macroscopic level; i.e. the large--scale phenomena we usually encounter in our daily experience. The edifice of what we call the classical physics, based essentially on Newton's laws of motion and Maxwell's electromagnetic theory of light, seemed impregnable. But then at the turn of the century, cracks in this structure started appearing.

P. A. M Dirac

Prelude to Quantum Mechanics

It is a matter of common experience that heated bodies radiate energy. They emit thermal radiation with a broad spectrum of wavelengths mainly in the infra-red region. If such radiation is contained inside a hollow cavity whose walls are opaque to the radiation and maintained at a constant temperature, then the energy distribution over various wavelengths becomes independent of the shape and composition of the cavity and is dependent only on the temperature--this is known as black-body radiation. Its properties can, therefore, be studied by means of the diffuse radiation coming out of a small hole from such a cavity. this was done in a series of experiments during 1985 - 1900, climaxed by the classic experimental studies due to Rubens and Kurlbaum in 1900, which established irrefutably the contradiction between experimentally observed energy distribution pattern in the black-body radiation and the prediction of classical physics. The correct formula for fitting the experimental data was guessed empirically by Max Planck and announced in a meeting of the German Physical Society on October 19, 1900. This later became famous as the Planck's law of black-body radiation.

All attempts to derive Planck's law from classical theory proved futile. This was a severe jolt to the complacence of classical physicists and from that point of time, the quantum theoretic revolution was on the anvil. On December 14, 1900 Planck presented a formal derivation on his law. As an "act of desperation" Planck was forced to assume that energy exchanges between matter and electromagnetic radiation take place only in the form of certain discrete packages (called the quantum); the energy content of each package is directly proportional to the corresponding frequency, the constant of proportionality being known as Planck's constant. This postulated existence of light quanta was standpoint of classical physics. Having let the spirit of quantum out of the bottle, Max Planck was himself scared of it and insisted on believing that packages of energy are involved only during emission and absorption of radiation and not during its propagation. To put it metaphorically, Planck's idea was that electromagnetic radiation is like butter, which can be bought or returned to the grocery store only in packages, although butter as such can exist in any desired quantity.

The first person to visualise that Planck's introduction of the idea of light quantum had wider significance was none other than Albert Einstein. In 1905, he took a bold step in extending Planck's idea by demonstrating that the concept of quantized bundles of electromagnetic radiation (these bundles are now called photons) propagating through space is essential for explaining the experimentally observed laws of photoelectric effect (emission of electrons from metallic surface irradiated by ultra-violet radiation). This seminal work paved way for the important realization that through a great weight

From a life of physics

RECALLING his exotic feelings on that momentous night in Heligoland island when he first became convinced of the mathematical viability of the new formulation of quantum mechanics he had just discovered, Heisenberg wrote: "At first, I was deeply alarmed. I had the feeling that, through the surface of atomic phenomena, I was looking at a strangely beautiful interior, and felt almost giddy at the thought that I now had to probe this wealth of mathematical structure nature had so generously spread out before me, I was far too excited to sleep, and so I made for the southern tip of the island, where climbing a rock jutting out into the sea, I waited for the sun to rise".

While introducing Werner Heisenberg at the ICTP, Trieste Symposium on Contemporary Physics in June 1968, Paul Dirac--himself one of the principal architects of quantum mechanics - made the following statement, which is an eloquent testimony to the humility and intellectual ethos pervading the creators of quantum mechanics: "I have the best of reasons for admiring Heisenberg. He and I were young research students at the same time, about the same age, working on the same problem. Heisenberg succeeded where I failed. There was a large mass of spectroscopic data accumulated at that time and Heisenberg found the proper way of handling it. In doing so he ushered in the golden age of theoretical physics, and for a few years after that it was easy for any second rate student to do first rate work".

Bohr and Einstein

of evidence had corroborated wave properties of electromagnetic radiation (compatible with Maxwell's electromagnetic theory), the wave theory of light has its limitations which are manifested in its inability to account for various phenomena involving interactions between matter and radiation, where the particle nature of electromagnetic radiation becomes applicable. Theoretical understanding of this apparently enigmatic wave-particle dual nature of electromagnetic challenge to the physicists of those days. Einstein had prophetically anticipated in 1909: "The next phase of the development of theoretical physics will bring us a theory of light that can be interpreted as a kind of fusion of the wave and particle theories". In fact, such a theoretical framework is precisely the key outcome flowing from the emergence of quantum mechanics.

Puzzles of Atomic Structure

Parallel to the development of the light quantum concept, another chain of concerted efforts was in full cry to unravel the atomic structure, which also revealed chinks in the classical theory. We know that atoms are the building blocks of all matter the universe is composed of. A major discovery a the dawn of this century, due to J. J. Thomson and his group at the Cavendish Laboratory. Cambridge, indicated that the atoms are not the indivisible constituent units of matter, instead they are rather complex systems formed by positively and negatively charged constituents. Thomson conjectured an atom to be formed by positively charged matter distributed uniformly charged electrons embedded in it, as are plums in a pudding.

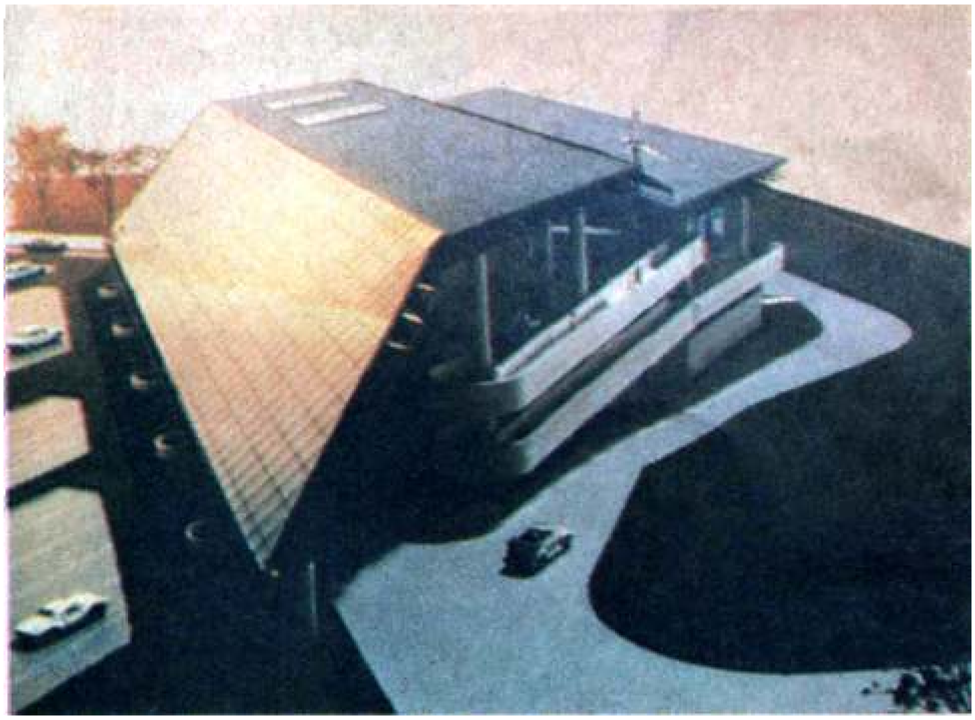
Then followed Ernest Rutherford's ingenious experimental studies at the University of Manchester (1911), which involved bombardment of atoms by high-energy projective known as alpha particles emitted by then newly discovered radoactive elements. To interpret the experimental results concerning scattering of these alpha particles, Rutherford inferred that the positive charge of atom is not distributed throughout its volume, but is concentrated in a very small region of diameter, say about 10^{-12} cm (this is known as nucleus of the atom), while the negatively charged electrons revolve about it at distances of the order of 10^{-8} cm. Due to Coulomb's law of electrostatic attraction operating between the nucleus and electrons, atomic electrons are expected to move round the nucleus along circular or elliptic orbits, just as do planets around the sun. Thus Rutherford's atomic model resembled the solar system and seemed quit attractive, but unfortunately there was a ticklish snag vitiating this model.

Max Born

According to classical theory of elecmagnetism, any accelerated electric charge is bound to emit electromagnetic radiation. It, therefore, follows that revolving atomic electrons must inevitably and rapidly lose all energy through radiation and eventually collapse into the nucleus. But atoms do exist! Thus one had to choose either the classical electromagnetic theory or the planetary model of atom. Physicists reluctantly chose the former. reluctantly, because Rutherford's experimental results could neither be disproved, nor explained without invoking the planetary model of atom. Moreover, doubts about the universal validity of classical electromagnetic theory had already started gaining ground in view of Planck-Einstein works on light-quanta concept. Later, reflecting on this state of confusion. Eisntein remarked:"It was exactly as if the ground was slipping away from under our feet and we had no firm soil that could build on."

It is this sort of chaotic and vacillating situation which provides the ideal soil for germinating revolutionary concepts in science. A brilliant intuitive attempt to resolve the atomic riddle came from Neils Bohr – a young Danish physicist. In 1913 Bohr proposed his famous theory which achieved synthesis of Rutherford's atomic model with Planck's light quanta hypothesis. The cardinal tenets of Bohr's theory were as follows:

(a) An atom possesses a number of discrete states in which no emission of radiation takes place. These states are fixed by certain ad-hoc rules and are known as 'stationary' states of the atomic system. Mechanical energy of atomic electrons is quantized in the sense that it can take up only certain discrete set of values, the intermediate values being prohibited by some yet undis-



A solar powered house

covered laws of nature.

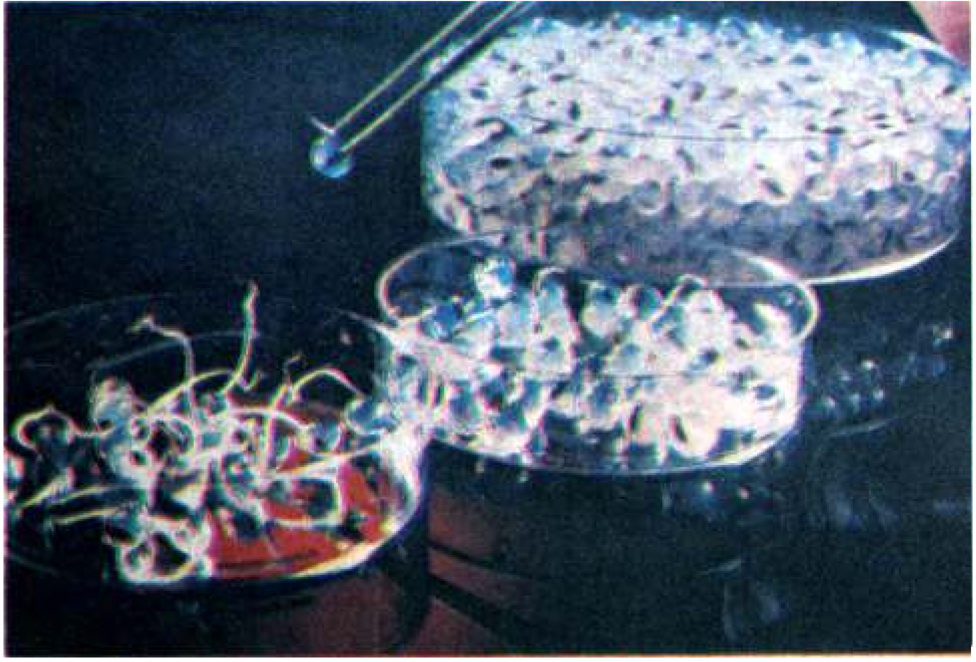
(b) Any emission or absorption of energy by an atom corresponds to the transition between two stationary states. Energy (E) of the light quanta emitted or absorbed during such a transition is determined by the relation $E = hn = A_1 -- A_2$ where A_1 A_2 are energies of the atom in the two stationary states, n is the frequency of radiation, and h is known as Planck's constant (10^{-27} erg-sec).

(c) The dynamical equilibrium of atomic systems in the stationary states is governed by the classical laws of mechanics, but the stationary state to another.

Prior to Bohr's theory, the empirical rules determining frequencies of the atomic spectral lines were regarded in the same light "as the lovely patterns on the wings of butterflies; their beauty can be admired; but they are not supposed to reVeal any fundamental biological laws." A major consequence of Bohr's theory was that these 'lovely patterns' of atomic spectroscopy became, for the first time, amenable to theoretical understanding. The characteristics of the atomic hydrogen emission spectrum fitted dramatically with the result derived from Bohr's theory.

Bohr's theory formed the crux of what is now called the 'Old Quantum Theory' which proved successful in interpreting a restricted region of atomic phenomena, particularly concerning one-electron atoms such as the hydrogen atom and helium ion. However, on close examination, these rules seemed to be a makeshift hodgepodge of ad-hoc principles and were strictly confined to only periodic or multiply-periodic motions. During 1922-1923, Max Born at Gottingen, in collaboration with his two young research students Wolfgang Pauli and Werner Heisenberg, critically investigated the validity of Bohr's theory for many-electron atoms such as the neutral helium atom. It was found that Bohr's theoretic approach, as applied to helium atom, does not yield results consistent with spectroscopic data. Born later reminisced: "We became more and more convinced that a radical change of the foundations of physics was necessary, i.e., a new kind of mechanics for which we used the term I quantum mechanics." However, I attempts to distill the principles of this unknown mechanics of the atomic phenomena out of a bewildering array of empirical facts were plagued with formidable difficulties. The state of despair among physicists of those days is poignantly reflected in the following I comment by Pauli (later destined to] become one of the most brilliant theoretical physicists of our time) on May 21, 1925: "Physics has run into a blind alley again. It has become too difficult for me, and I would prefer to be a movie comedian or something similar and hear no more about physics." Just then close on the heels came the epochal contribution by Heisenberg which illuminated the way out I from this impasse.

Synthetic lettuce seeds, produced using recombinant DNA technology, beginning to germinate

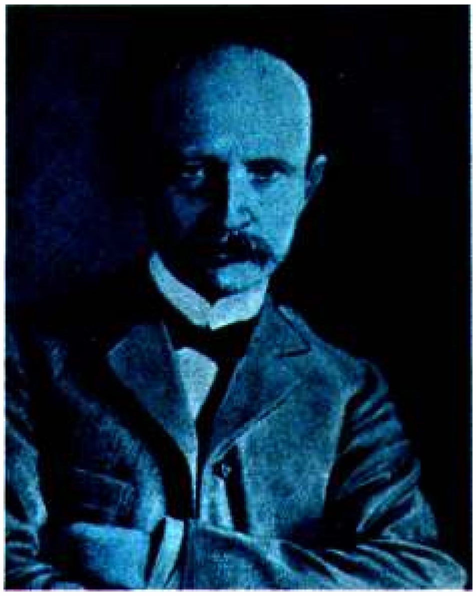


Birth of Quantum Mechanics

In science it is impossible to open up a new vista unless one is prepared to leave the safe anchorage of established doctrines. It is this spirit which sustained the venture culminating in the discovery of quantum mechanics. An important step in this direction was the realization that the Bohr theory of dealing with atomic phenomena is an unsatisfactory hybrid of quantum concepts grafted on to classical mechanics. As long as the classical picture of well defined particle orbits was retained, it remained conceptually incomprehensible as to why only certain electronic orbits should be allowed in the atom. This prompted Heisenberg to suggest the viewpoint that particle states at the atomic level are not describable in terms of well-defined orbits, instead a radically different mathematical framework was needed.

Towards the end of May, 1925 Heisenberg—down with hay fever-went to Heligoland for convalescing. There he got the first inkling of a broad outline of this new mathematical scheme on the basis of which he succeeded in deriving correct energy expressions for harmonic and inharmonic oscillator and a simple rotating system. However, Heisenberg was himself sceptical about the rules of the game involved; particularly because the algebra he used implied that the multiplication of two quantities a x b is not necessarily equal to b x a (this is technically called 'noncommutative algebra'). Heisenberg later reminisced: “1 felt that this non-commutativity was a major disturbing point of difficulty in the whole scheme. The choice, as it seemed to me then, was to either complete the work quickly or throw it into the flames.”

However, after receiving favourable response from his invaluable critic-friend Pauli, Heisenberg was inspired to complete the work and he submitted the paper to his supervisor Born. Walking down the memory lane, Born writes in his Recollections: “After having sent Heisenberg's paper for publication, I began to ponder about his strange multiplication rule. It did not give me any rest... And one morning I suddenly saw light—I recalled an algebraic theory which I had learnt in my student days." This 'algebraic theory' concerns matrices—mathematical entities which are two-dimensional array of numbers—whose non-commutative algebra had been formulated by Arthur Cayley around 1850. Prior to 1925, matrices had rarely been used by physicists. Born identified Heisenberg's multiplication rule as that pertaining to multiplication of two matrices: it then became transparent that a salient feature of the new scheme was that observable physical quantities are represented by matrices.



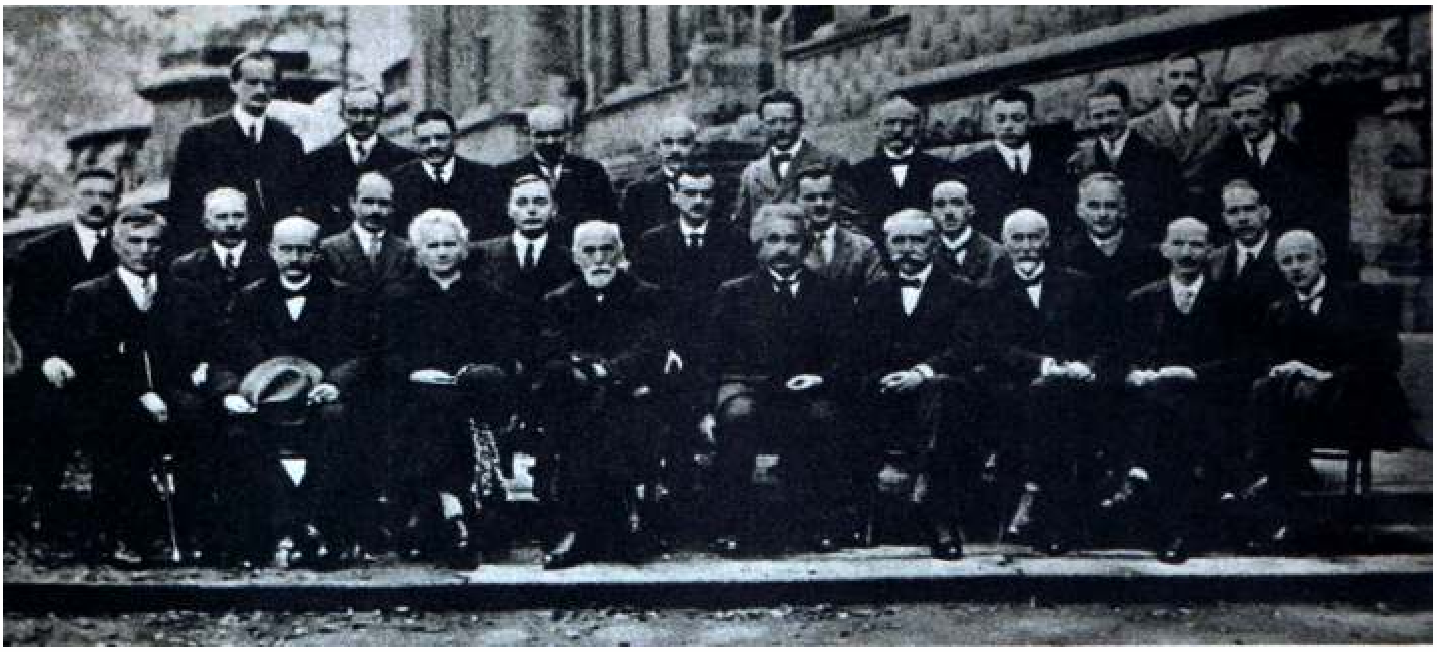
Max Planck

Interestingly, Born himself was not very proficient in handling matrices; he faced considerable difficulties in providing the necessary mathematical refinement to Heisenberg's ideas. At that point of time, it happened that Born, while travelling by train from Göttingen to Hannover, was talking to one of his colleagues about the problems bothering him about matrices. A fellow passenger was Pascual Jordan, a bright research student well-trained in the manipulation of matrices, who overheard this piece of conversation. Jordan introduced himself to Born and offered to assist Born. That was the genesis of the fruitful collaboration

Uncertainty principle

DURING the spring of 1926, Heisenberg visited the University of Berlin and there he had threadbare discussions with Einstein concerning the central ideas of the then newly discovered quantum mechanics. Heisenberg drew Einstein's attention to the epistemological principle underlying his formulation of quantum mechanics, viz. a physical theory must deal only with directly observable entities. To reinforce this viewpoint, Heisenberg told Einstein: “Isn't that precisely what you have done with relativity? You argued that it is impermissible to speak of absolute time, simply because absolute time cannot be observed". Einstein surprised Heisenberg by replying: “Possibly I did use this kind of reasoning, but it is nonsense all the same. Perhaps I could put it more diplomatically by saying that it may be heuristically useful to keep in mind what one has actually observed. But on principle it is quite wrong to try founding a theory on observable quantities alone... It is the theory which decides what we can observe."

It is this last remark of Einstein that stimulated Heisenberg to reflect on the following question—Does quantum mechanical theory imply any constraint on the measurability of physical quantities? This culminated in the discovery of the uncertainty principle, which is now recognised as one of the cornerstones of the conceptual structure of quantum mechanics. In tune with the spirit of Einstein's comment. Heisenberg inferred from the mathematical formalism of quantum mechanics that whatever best experimental arrangement is designed, inner consistency of quantum mechanical rules demands that it will not be possible to circumvent the limit on accuracy of simultaneous measurement as given by the uncertainty principle. Quantum mechanics does not permit the concept of trajectory of a particle because it presumes exact simultaneous knowledge of position and momentum. If Anyone can figure out a way of measuring the position and momentum at any instant with accuracy exceeding the limit embodied in the uncertainty principle, quantum mechanics would be invalidated. Thus the uncertainty principle ‘protects’ quantum mechanics.



Solvay Conference of 1927: First row (from left): I. Langmuir, M. Planck, Madame Curie, H. A. Lorentz, A. Einstein, P. Langevin, Ch. E. Guye, C. T. R. Wilson. O. W. Richardson. Second row: P. Debye, M. Knudsen, W. L. Bragg, H. A. Kramers, P. A. M. Dirac, A. H. Compton, L. V. de Broglie, M. Born, N. Bohr. Third row: A. Piccard, E. Henriot, P. Ehrenfest, Ep. Herzen, Th. de Donder, E. Schrödinger, E. Verschaffelt, W. Pauli, H'. Heisenberg, R. H. Fowler, L. Brillouin.

between Born, Heisenberg and Jordan. By the end of 1925, their concentrated efforts gave rise to a rigorous and coherent mathematical framework which promised to embrace all the multifarious facets *of* atomic and subatomic physics (this is known as the matrix mechanics version of quantum mechanics).

It was quickly followed by an avalanche of breath-taking developments: P.A.M. Dirac from Cambridge, England independently formulated the pivotal ideas of matrix mechanics in a remarkably elegant and logically systematic way; Pauli demonstrated that the energy spectrum of hydrogen atom can be successfully explained in all its details by invoking the rules of the new mechanics, in a series of four strikingly original papers published in the early half of 1926, Erwin Schrodinger from University of Zurich propounded an entirely different version of quantum mechanics (known as wave mechanics). Whereas Heisenberg's method is algebraic, Schrodinger's formalism hinges on a differential equation whose solutions furnish results in agreement with matrix mechanics. The merit of Schrodinger's scheme lies in the fact that it is amenable to more tractable treatment, which is otherwise very much involved in the matrix algebraic framework. Mathematical equivalence between wave mechanics and a matrix mechanics was conclusively proved by Schrodinger. In the words of the well-known physicist George Gamow: "Discovery of two different formulations of quantum mechanics is like as if America was discovered by Columbus sailing westward across the Atlantic Ocean, and by some equally daring Japanese sailing eastward across the Pacific Ocean".

Epistemological Aspects

What is quantum mechanics all about? Well, to put it succinctly, quantum mechanics is a systematic physical theory of atomic and sub-atomic phenomena, based on a set of self-consistent mathematical rules supplemented by appropriate physical interpretation. It is important to emphasize that entities on an atomic scale do not behave like anything that we have direct experience about. To quote Heisenberg: "Since atomic and subatomic entities look like wave on one occasion and like particle on the next, we must obviously come up with new concepts. Perhaps, one ought to call such entities 'wavicles', mathematical description of their behaviour being provided by quantum mechanics."

Werner Heisenberg

Advent of quantum mechanics also firmly established that the principle of determinism does not operate at the level of atomic and sub-atomic phenomena. At the outset, let us explain in simple terms what is meant by determinism—if relevant initial conditions are known, one should be able to predict the future happenings precisely. Classical mechanics is deterministic in the sense that if the state of a particle, characterized by its position and velocity, is accurately specified at any instant, the state at any other instant can be exactly estimated using the laws of mechanics. This mechanical determinism gradually became an article of faith and got elevated to the status of a philosophic principle for the entire gamut of exact science concerning macroscopic phenomena.

However, quantum mechanical formalism is inherently probabilistic in character. The basic equations of quantum mechanics do not represent actual motion of particles (like Newtonian equations of motion); they essentially determine the possible states of an atomic system. On the basis of these equations, one can predict, in general, only the statistical probability of experimental outcomes. Suppose we have an Atom in an excited state, which is going to emit light. Using quantum mechanics one cannot predict exactly at which instant it would emit light— we can only estimate the probability of emission at any particular instant. This feature is technically called 'indeterministic character' of quantum mechanics.

It is this probabilistic aspect—so inextricably linked with quantum mechanical description of microscopic physical phenomena—which appeared repugnant to some of the brilliant luminaries in physics, the most prominent of them being Einstein. After the historic Solvay Conference at Brussels, during the autumn of 1927, where *q* most of the founding fathers of quantum theory gathered and finally thrashed out the theoretical foundations of quantum mechanics, Einstein put forward his contention that "though quantum mechanics is free from logical inconsistencies, it can at best be an incomplete description of physical reality," a deep and unshakable conviction which he clung to for the rest of his life. Einstein's endeavour was to drive home the point, that quantum mechanics does not provide complete description of an individual micro-object.

It is, of course, not unnatural to inquire whether in the realm of atomic phenomena there is another deeper stratum to reality where determinism operates and statistical results of quantum mechanics are essentially due to averaging over the hypothetical extra variables, called 'hidden variables', which would supplement quantum mechanical description of the state of a system. This approach has sparked off attempts towards formulating the so-called hidden-variable theory which, apart from reproducing all known experimentally verified quantum mechanical results, would provide definite predictions about individual results of observation. Such a theory, it is hoped, will not only restore determinism to the domain of microphysics, it would also dispense with the peculiar dichotomy of physics into microscopic and macroscopic phenomena and re-establish a unitary account of the physical world, a tantalising prospect which allured many a celebrated physicist, Louis de Broglie and David Bohm among them.

Erwin Schrodinger

Neils Bohr

Ernest Rutherford

Epilogue

"The point is not to pocket the truth, but to chase it"—Elio Vittorini, Italian sociologist.

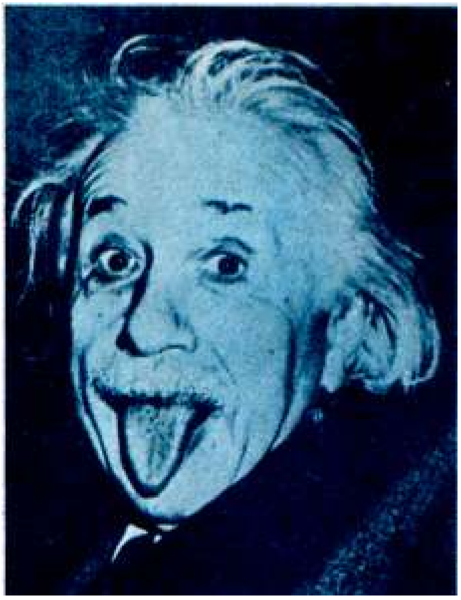
Historiography of science usually highlights only the magical insights, or the revolutionary leaps of a Newton or an Einstein. But proper appreciation of these success stories is impossible without getting to know how hard the background struggle is—how easy it is to be led astray, how complex it is to visualise what the next step should be. History of the development of quantum mechanics bears ample testimony to the fact that discovery of scientific truth is not merely a singular stroke of genius by an isolated individual—it is a rather slow, groping, painstaking process evolving through interaction of many minds.

Physics of atomic phenomena has also underscored the pitfalls involved in blind extrapolation of ideas from one domain to another and our experience with micro-domain has taught us how little obligation nature has to conform to our a priori intuitive conceptions.

Futility of trying to accommodate qualitatively new facts in an older framework of pre-conceived notions is most strikingly manifested in the dogmatic adhesion to traditionally rigid doctrines of classical physics by many outstanding physicists, resulting in their inability to grasp the germane nuances of quantum mechanics. An old metaphor may be relevant here—if we pour new wine into an old bottle, it misleads us into occupying ourselves with the cracks in the old bottle, rather than rejoicing over the new wine.

Over the past sixty years, the mathematical formalism of quantum mechanics has been repeatedly checked for its predictions. To date, there has been complete agreement with experimental results for all known microsystems and their interaction with radiatioin. Quantum mechanics has been also found indispensable to account for the behaviour of macrosystems such as superconductors, superfluids and lasers. Even for more common macrosystems, like solids, it is quantum mechanics which has to be applied to calculate their various phenomenological properties.

However, the interpretational aspect and conceptual structure of quantum mechanics still continue to present an array of problems which are both fascinating and puzzling. In particular, there are various ticklish foundational issues which cry out for deeper understanding, for example, quantum mechanics does not provide completely satisfactory explanation of certain non- intuitive and paradoxical aspects concerning measurement process performed on a system to register its physical attributes. The interaction between system and measuring apparatus and system gets affected due to this interaction are not yet amenable to rigorous and unambiguous quantum mechanical treatment. Conflicting viewpoints also persist about the interesting question whether quantum mechanical principles are universally valid for macroworld and at sufficiently high energies for microsystems; a careful and comprehensive investigation is needed to settle this issue.



Albert Einstein

Albeit, reconciliation of quantum mechanics with special theory of relativity has been achieved to an appreciable extent, a few conceptual and mathematical niceties remain to be clarified. Again, the merger between quantum mechanics and general theory of relativity (Einstein's theory of gravity, which is based on the idea that gravitation is essentially an effect of the curvature of the space-time continuum) is beset with much more intractable difficulties. Nevertheless, formulation of quantum mechanical theory of gravity is now assuming increasing importance for the purpose of unifying gravity with other forces of nature. This effort may itself call for as drastic change of ideas as warranted by quantum mechanics or relativity separately.

Past experience reveals that the refinement of the structural precepts of the foundations of physics can play a "decisive role in moulding the evolution of our understanding of the physical world. Often these analyses require considerable time to generate any significant impact. The mounting interest in the foundational problems of quantum mechanics is perhaps accentuated by the fact that quantum mechanics has started reacting on its own foundations and it is now becoming necessary to pinpoint more precisely the delicate questions of fundamental principles, coupled with their possible empirical relevance. Who knows, during the next sixty years of quantum mechanics, there may be major surprises in the offing! □

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Clash of giants

DURING the autumn of 1927, in the Solvay Congress at Brussels, attended by almost all the celebrated physicists of that era, vigorous argumentation took place between Einstein and the quantum physicists headed by Niels Bohr, centering around the pivotal concepts of quantum mechanics; in particular, the validity of uncertainty principle. We reproduce below excerpts from Heisenberg's reminiscences about this exciting tussle involving some of the finest intellects science has ever produced:

"The discussion usually started at breakfast, with Einstein serving us with an imaginary experiment by which he thought he had definitely refuted the uncertainty principle. We would at once examine his fresh offering and as a rule, by suppertime, Bohr could prove to Einstein that his thought experiment failed to shake the uncertainty principle. Einstein would look a bit worried, but by next morning he was ready with a new imaginary experiment, more complicated than the last. This attempt would fare no better by evening, and after the same game had been continued for a few days, Einstein's close friend Paul Ehrenfest (a leading physicist from Leyden in Holland) said: “Einstein, I am ashamed of you; you are arguing against the new quantum theory just as your opponents argue about relativity theory”. But even this friendly admonition went unheard. Once again, it was driven home to me how terribly difficult it is to give up an attitude on which one's entire scientific approach and career have been based."

Reflecting on this debate, Einstein himself remarked a few years before his death: “Of course, I might have been wrong, but perhaps I have earned the right to make my mistakes.”